

The Secret of Hair Whorl

发旋的奥秘

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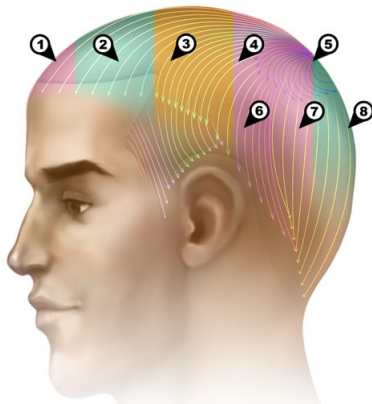


Figure 1: A baby with possibly two source points and one saddle point on his head at the same time (left). Hairs on boundary of hair region on your scalp always point outward (right)

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Topological Type

All oriented compact surfaces can be classified by their genus g and number of boundaries b . Therefore, we use

$$(g, b)$$

to represent the topological type of an oriented surface S .

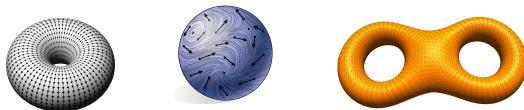


Figure 2: tangent vector field of torus, sphere and bi-torus

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Poincaré-Hopf Index Theorem

Assume S is a compact, oriented smooth surface, v_S is a smooth tangent vector field with isolated zeros $\text{Zero}(v_S)$. If S has boundaries^a, v_S point along the exterior normal direction^b, then we have

$$\sum_{p \in \text{Zero}(v_S)} \text{Ind}(v_S, p) = \chi(S)$$

where $\text{Ind}(v_S, p)$ is the index of zero p , and $\chi(S)$ is the Euler-Poincaré characteristic.

^afor a surface S with genus g and number of boundaries b , the Euler-Poincaré characteristic is

$$\chi(S) = 2(1 - g) - b$$

^bfor any point $p \in \partial S$, $v_S(p) \cdot \mathbf{n}(p) > 0$

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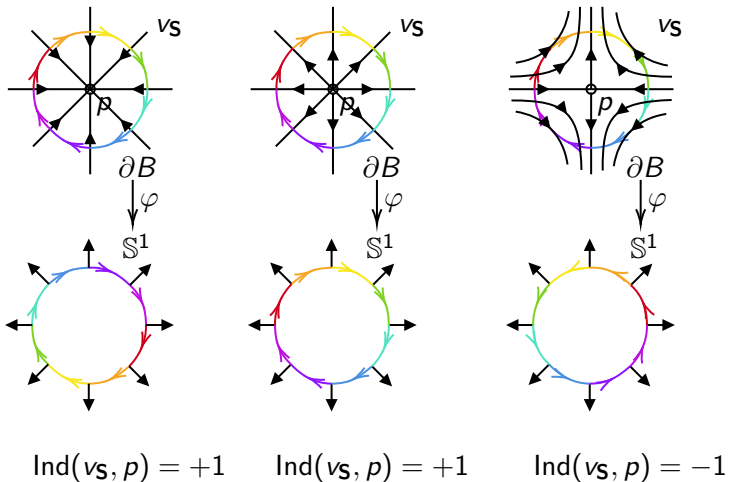


Figure 3: index of sink (left), source (middle) and saddle (right) point is +1, +1, -1, respectively

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We now show that the hair whorl over your head is guaranteed by Poincaré-Hopf index theorem.

“Hair Whorl” theorem

- The hair region of human scalp can be considered as a smooth, compact, oriented surface with a boundary, so its Euler-Poincaré characteristic is 1
- The hairs, with their projective directions, can be considered as an unit tangent vector field
- The hair whorl, can be considered as the zero point of the vector field
- Assuming that the hairs on boundary always point along the exterior normal direction

$$\sum_{p \in \text{Zero}(v_S)} \text{Ind}(v_S, p) = 1$$

Everyone has at least one hair whorl.

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